# MODELING THE PERFORMANCE OF THE HUMAN (PILOT) INTERACTION IN A SYNTHETIC FLIGHT DOMAIN: INFORMATION THEORETIC APPROACH

Celestine A. Ntuen

Human-Machine Systems Engineering Lab.

Department of Industrial Engineering

North Carolina A&T State University

Greensboro, NC 27411

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# Information Theoretic Models of Human-Machine Interaction

#### ABSTRACT

Current advances in computing technology are devoid of formal methods that describe the theories of how information is shared between the humans and machines. Specifically, in the domain of human-machine interaction, a common mathematical foundation is lacking. The aim of this paper is to propose a formal method of human-machine (H-M) interaction paradigm from information view point. The methods presented are interpretation - and context - free and can be used both in experimental analysis as well as in modeling problems.

#### 1. INTRODUCTION

The effectiveness of modern information technology depends in parts on the level of human-machine interaction. The human users of information systems (softwares) are faced with information state space which are complex. This complexity evolves around both human behavior and the machine state dynamics (see, e.g; [2,7]). Unfortunately, as many studies [11,14] indicate, the level of information loading continue to be the number one problem affecting the design of softwares. One reason to this problem is that software engineers and information scientists seem to ignore the formal approach to the design of H-M interface in the software development life cycle.

Suffice to say that even in a simple human-computer system environment, the issue of developing a formal method (mathematical theory) of interface paradigm still remains an enigma (see, e.g; [1,5,6]). Rasmussen [14] supports this view by observing that "in human-machine interaction, it appears to be necessary to consider the same distinction between signals and signs for the significance of human acts as it is for the information observed by a human. This dynamic interaction with the environment of complex behaviors calls for a very efficient feature

extraction and classification and dynamic coordination of the human-machine system with the task environment.

Most existing formal methods of H-M interaction are context specific and concentrate more on:

- (a) the allocation of tasks to human operators and machines [1,5,10,19];
- (b) display design and information presentation theories [1];
- (c) communication bandwidth and dialogue protocols [13,15];
- (d) group behavior theory [4,18];
- (e) matching human behavior maps to information load [13,17].

The citations above have a common drawback in that no general method of H-M interaction exists. What is often described is the engineering process of H-M interaction which lacks the rigorous scientific theories. Methodologically, information theory is needed to characterize the H-M interaction environment. This problem is presented here in a context - and interpretation free format. The discussions are based on elementary functions of automaton.

## 2. PRELIMINARIES

The human-machine interaction (HMI) problem can be stated succinctly as follows: given a computer system C, and the human (as a controller, supervisor, user, etc.) H, we are interested in the design d; such that

$$d = \phi_H \wedge \phi_c \neq 0$$

and 
$$D = \phi_H \bigcup \phi_c$$

We use D to be the universe of design discourse; d to typify the interaction domain such d  $\epsilon$  D;

 $\phi_{\rm H}$  and  $\phi_{\rm c}$  are the feature space characterizing the human and computer systems respectively. When the word "model" is used, we shall mean the elements of the computer system. Thus,  $\phi_{\rm c}$  is a model feature space whereas  $\phi_{\rm H}$  is a physical feature space. We also define the general feature space  $\phi$  by the three element grammar defined by

$$\phi = \{I, \lambda, P\}$$

where I is the information vector characterized by the four tupple

$$I = \{S, M, U, V\}$$

with S as the information source (or sensory matrix); M the information modality which assigns "type" (logical, numerical, etc.) to the value of the information; U is the information control vector that triggers information occurrence; and V is a matrix of input-output data defined by

$$V := \langle I^i \otimes I^o \rangle$$

 $I^i$  is the input data usually from the physical source (user-input) and  $I^o$  is the output data, usually from the model source (computer system). The operator " $\otimes$ " is defined as concatenation, operator, e.g;  $\{a\} \otimes \{b\} = \{ab\}\lambda$  defines the global state of the H-M system and is defined by

$$\lambda$$
: =  $\langle T \otimes E \rangle$ 

where T is the task vector and E is a environment disturbance. P is a performance matrix defined by column-wise concatenation operator | over the tupple elements defined by

where  $\delta$  is a Mealey automaton state-transition function [13] defined by

$$\delta: \lambda \times v_i \rightarrow I^I$$

 $\omega$  is a Mealey automaton output function defined by

$$\omega$$
:  $\lambda \times v_2 \rightarrow I^o$ 

 $\beta$  is a combination network on  $\delta$  and  $\omega$  defined by

$$\beta: \delta \times \omega \rightarrow \beta$$

$$V_{1} = \left(\frac{\partial T}{\partial V}\right) \left[\frac{dV}{dI^{o}}\right]$$

$$V_{2} = \left(\frac{\partial T}{\partial V}\right) \left[\frac{dV}{dI^{i}}\right]$$

Note that  $\frac{\partial T}{\partial V}$  uniquely defines the differential change in the task information with respect to

input-output matrix V. For a example, in a supervisory control task, this differential may be a change in the domain of diagnostic problem solving such as reading pressure or temperature gauges.  $dv/dI^i$  is the qualitative change in output data assuming no new input data

# 3. FORMAL DESCRIPTION

The HMI system is described by the following sets:

Terminal-state function

$$Y: = \alpha_d(I, P, Z)$$

where  $\alpha_d$  is a translation function mapping the features  $\phi_H$  and  $\phi_c$  in feature space d  $\epsilon$  D.  $\alpha_D$  is a many-to-many corresponding mapping with P as the evaluation function.

$$Z = \left\{ Z_1, \beta, Z_2 \right\}$$

where Z<sub>1</sub> is the physical task vector defined by

$$Z_1$$
: =  $\omega$  (  $I$ ,  $\lambda$  )

Z<sub>2</sub> is the model for information combination defined by

$$Z_2$$
:  $\delta$  (I, $\lambda$ )

**DEFINITION**. The interaction is said to be "symbiotic" optimal if

$$\{ \forall c' \in \phi_c; \exists h \in \phi_H \} \phi_h \land \phi_{c'} = \phi_{c'}$$

The algebraic relation is that for every model feature  $\epsilon$   $\epsilon$   $\phi_c$ , the human can interact successfully to perform a defined task. The concept of symbiosis is to measure the level of cooperation between the physical and model elements. This relation can be proved easily by invoking the laws of absorption which argues that if  $\phi_{\epsilon} \subseteq \phi_h$ , then  $\phi_{\epsilon} \land \phi_h = \phi_{\epsilon}$ , where  $\wedge$  is a conjunction operator.

**DEFINITION.** The performance matrix is a linear manifold structure of Z. This property is a fundamental approach to information aggregation. Note that  $Z = \{Z_1, Z_2\}$  represents information structure associated with the physical (human) and the model (computer) elements. If the event, say  $h \in H$  occurs with observation error  $e_h$ ; and the event say  $c \in C$  occurs with model error  $e_c$ . By definition,  $Z = \{Z_1 \rightarrow Z_1 + e_h, \beta, Z_2 \rightarrow Z_2 + e_c\}$ .

Since the systems is considered to be dynamic, this allows us to write, Z as a time dependent system of control automation:

$$\dot{Z} = A Z + E \\
P = G Z$$

where A is the matrix derived by  $\delta \parallel \omega \parallel \beta$ , E is the error matrix derived by concatenation of  $e_h$  and  $e_{\epsilon}$ , and G is a constant performance matrix. Note however that A and G are chosen to be semi-positive definitive and the values of Z are obtained via real time observation. An

example is the human pilot interacting with the pilot associate program in deciding on where to land an aircraft during a severe storm.

**DEFINITION.** Let  $\phi$  be an Euclidian information space. Consider a subspace  $\phi$ , such that  $\phi_s \wedge \phi = 0$ . Then  $\phi$  can be represented in the form

$$\phi = \varphi(z) + N$$

where  $z \in \phi_s$ ,  $\varphi(z) \in \phi_s$  and N is orthogonal to  $\varphi(z)$ . The property of  $\phi$  is such that

$$E \{ \varphi(z) + N \} = E \{ \phi - \varphi(z) \} \cdot \varphi(z) \} = 0$$

Further, the distance between  $\phi$  and any point p in  $\varphi(z)$  satisfies

$$E\left\{(\phi-p)^2\right\} \geq E\left\{(\phi-\varphi(z)) \cdot \varphi(z)\right\} = 0.$$

with the equality if  $p = \varphi(z)$ ;  $\varphi(z)$  is known as the projection of  $\varphi(z)$  on  $\varphi(z)$ . This definition stipulates the relationship between the human observer trying to project his or her corporal self into the domain of a model state space. An example of  $\varphi(z)$  is a pilot undergoing a flight handling simulation exercise and  $\varphi(z)$  is the model information characterizing the aircraft dynamics. The orthogonal vector N may represent the actual observation data during the experiment.

DEFINITION Let r(D) be a measure of H-M interaction design effectiveness. Then we define

$$r(D) = \frac{Min \{(\phi_c \land \phi_h), \phi_h \ 0\}}{Max \{\phi_c, \phi_h\}}$$

PROPOSITION. Let  $r(\phi_c)$  and  $r(\phi_b)$  represent the design effectiveness of model and human elements, then  $r(\phi_c \land \phi_b) \le r(\phi_c) + r(\phi_b)$ 

Proof. The result above follows the triangle law of inequality and the law of conjunction operator.

**DEFINITION**. Let m(D) be a measure of H-M interaction design efficiency. Then we define

$$m(D) = \frac{\omega(I_h, \lambda_h)}{\max\{Z_1, Z_2\}, \text{ for all } h \in H}$$

Note that efficiency is used here to measure the human elements that have been tested and validated for the system.

**DEFINITION.** Let s(D) be a measure of interaction "symbiosis" between H and C. Then s(D) is related to r(D) and m(D) by s(D) = r(D)/m (D);  $s(D) \ge 0$  and m (D)  $\ne 0$ .

Note that if s(D) = 0 then r(D) = 0 implies that  $\phi_c \wedge \phi_h = 0$ .

**DEFINITION**. Let  $\alpha_D$  be an information mapping function on the universe of design discourse D such that the probability  $\wedge$  (D) exists.  $\wedge$  (D) follows the usual definition of probability axioms, such that

$$\sum_{d \in D} \Lambda(D) = 1.$$

We can therefore define the mapping function  $\alpha_D(\phi_c, \phi_b)$  by the relation

$$\alpha_D(\phi_c, \phi_h) = \min\{ \overline{\Lambda}(\phi_h), \overline{\Lambda}(\phi_c), \overline{\Lambda}(\phi_h \Lambda \phi_c) \}$$

**DEFINITION.** Assume that information value can be measured on some distance metric  $n(\phi)$ . Further, assume the existence of optimal policy

$$\phi^* \in \phi_h \wedge \phi_c \quad \forall h \in H; \ \forall c \in C.$$

Define the design error  $e_d$  by  $e_d = \phi_h - \phi_c$ .  $e_d$  can be written in terms of  $\phi^*$  by  $e_d = (\phi_h + \phi^*) - (\phi_c - \phi^*)$ . If there are d design variables observed in  $\phi$ ; then the d - norm error distance  $n(\phi)$  is defined by

$$n(\phi) = ||d||$$
 , that is

$$n(\phi) = \left[ (\phi_h + \phi^*)^d + (\phi^* - \phi_c)^d \right]^{1/d}$$

**PROPOSITION.** If  $\phi^* - \phi_c = 0$ , then the distance measure  $n(\phi)$  is said to be regular with respect to the human observer. In this case, the human is said to "gain" all the information in  $\phi_c$ 

Proof. If  $\phi^* - \phi_c = 0$ ; then  $\phi^* = \phi_c$ . By definition,  $\phi^* = \phi_h \wedge \phi_c$ 

that is  $\phi_c = \phi_h \wedge \phi_c$ . By rules of Boolean algebra;  $\phi_c \subseteq \phi_h$ ; and  $\phi = \phi_h$  is the universal set. Therefore  $\phi \wedge \phi_c = \phi_c$ . Hence,  $n(\phi) = \{(\phi_h + \phi^*)^d\}^{1/d} = \phi_h + \phi^*$ ; this implies that  $\phi^* = \phi_c$  is the gain. An example of this proposition is used in developing decision support systems. Here,  $\phi_h$  is what the person using the system had known already,  $\phi^* = \phi_c$  is the decision support information from the computer which is new to the human. If at the end of interaction, the

**PROPOSITION.** Let  $H_I$  be experimental or observation matrix which is a positive definite. Then  $H_I = \phi_h$ .  $\phi^{-1}_c$ 

human has learned all  $\phi_c$ , by the proposition, information gain has taken place.

Proof. Let the matrix function  $H_1$ :  $R \circledast J \to R$  be induced in the natural way by multiplying the design matrix R by a unit matrix J. R is defined such that

R  $\oplus$  J: =  $\langle \phi_h, \phi_c, J \rangle$ ,  $\forall h \in H, \forall c \in C$  with the definitions:

 $\phi_{\mathtt{h}}:=\phi_{\mathtt{h}} \oplus \mathrm{J}_{\mathtt{h}} \twoheadrightarrow \phi_{\mathtt{h}}$ 

 $\phi_{\rm c}$  : =  $\phi_{\rm c} \oplus J_{\rm c} \rightarrow \phi_{\rm h}$ 

 $J_c = J_h \parallel J_c$ , when "\|" means column wise concatenation. Without loss of meaning. Let us assume the relationship:  $J_h$  and  $J_c$  to be unit matrices defined on  $\phi_h$  and  $\phi_c$  respectively.

$$\phi_h \cdot \phi_c \cdot \phi_c^{-1} = \phi_h^{-1} (\phi_h \cdot J_h \phi_c J_c) \phi_c^{-1} \phi_h^{-1} J_h$$

The left hand side of equation is equal to  $\phi_h \cdot J_c$ . And the right hand side is simplified to

 $J_h \cdot \phi_c \cdot H_I \cdot . \quad \text{Thus, } \phi_h \cdot J_c \, = \, J_h \, \phi_c \, H_I \, J_h$ 

If we post concatenate  $J_h$  on both the left and right hand side of the equation above, we have

$$\phi_{h} \cdot J = J_{h} \phi_{c} H_{I} J$$

$$\phi_{\rm h} = J_{\rm h} \, \phi_{\rm c} \, H_{\rm I}$$

Hence 
$$H_I = \left(\phi_h \cdot J_h^{-1}\right)\phi_c^{-1}$$

$$H_I = \phi_h \cdot \phi_c^{-1}$$
 (since  $J_h^{-1} = J_h$ )

Note that the model information matrix has become a weighted matrix for the observation matrix. We assume that  $\phi_h$  and  $\phi_c$  have the same cardinality.

#### 4. CONCLUSIONS

The development of information theoretic models based on abstraction and automaton theory, provides a framework for measuring the effectiveness and efficiency of human-machine interaction design. In addition, a general framework for formal methods of modeling H-M interaction is suggested.

As a prolegomenous discussion, the basic definitions and some propositions with proofs are presented. Specifically, the formal descriptions rely more on abstractions and equivalence formulations of formal method rather than inductive hypothesis. The presentation is open-ended in format. Thus, the concept presentation are useful in disciplines such as software engineering, fuzzy models, and decision support system (expert system) techniques.

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